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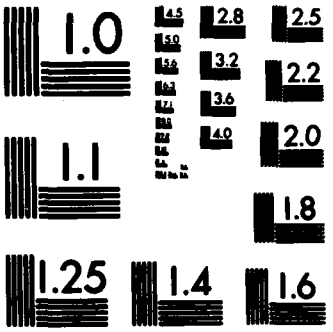

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MOMENT ESTIMATORS OF DEFECT PATTERN FREQUENCIES, ALLOWING FOR INSPECTION ERROR

by

Norman L. Johnson and Samuel Kotz

University of North Carolina
Chapel Hill, North Carolina

University of Maryland
College Park, Maryland

ABSTRACT

Unbiased estimators of the number of individuals in a lot possessing various patterns of types of defects are constructed. Explicit formulas are given for cases of two and three types of defect. Application of the formulas requires knowledge of the probabilities of various kinds of errors in the inspection process.

Key Words & Phrases: Inspection errors; Estimation; Quality control; Sampling inspection.

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1. Introduction

In [1] [2], we introduced the term *defect pattern*, to reflect the incidence of defects of different types in the same individual. If there are $\underline{k}$ types of defect, in all, under consideration, then an individual's defect pattern is represented by a binary (0,1) sequence of $\underline{k}$ digits, with the j-th digit equal to 0, or 1 according as the j-th type of defect is not, or is, present in the individual. There are 2^k possible different defect patterns.

We will denote the defect pattern $(g_1,g_2,\ldots,g_k)$ by $(\underline{g})$, and the number of individuals, in a lot of size N, possessing defect pattern $(\underline{g})$, by $D_{\underline{g}}$. Clearly

$$\sum_{(\underline{g})} D_{\underline{g}} = N$$

where $\sum_{(\underline{g})} \equiv \sum_{g_1=0}^{1} \sum_{g_2=0}^{1} \cdots \sum_{g_k=0}^{1}$ denotes summation over all 2^k patterns.

If a random sample of size n is chosen (without replacement) from the lot, and $Z_{\underline{g}}$ of these are found, on inspection, to have defect pattern $(\underline{g})$, then, if inspection is perfect a natural (and unbiased) estimator of $D_{\underline{g}}$, obtained by equating observed and expected values of $Z_{\underline{g}}$, is $NZ_{\underline{g}}/n$. If inspection is imperfect, the situation is not so simple, but the same method can be used, provided the probabilities of correct and incorrect assignments of defect patterns are known. Since the resultant estimators - $\tilde{D}_{\underline{g}}$, say - are obtained by equating first moments of sample and population values of the $Z_{\underline{g}}$'s, we will call them *moment estimators*.

We will suppose that

(i) if a defect of type j is present, the probability that it will be detected is p_j;

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(ii) if a defect of type j is not present, the probability that it will be (erroneously) detected is p_j';

and

(iii) for any defect pattern, *decisions* in regard to presence or absence of each type (or disjoint sets of types) of defect are mutually independent.

This last assumption may not always be justified, but it serves as a useful point of reference.

With assumptions (i)-(iii), the probability that an individual with defect pattern ($\underset{\sim}{f}$) is classified, as a result of inspection, as having pattern ($\underset{\sim}{g}$) is

$$p_{\underset{\sim}{g}|\underset{\sim}{f}} = \prod_{j=1}^{k} \{p_j^{g_j f_j} p_j'^{g_j(1-f_j)} (1-p_j)^{(1-g_j)f_j} (1-p_j')^{(1-g_j)(1-f_j)}\} \qquad (1)$$

We will further assume that $p_j > p_j'$ for all j. This does not affect many of our formulas, and it is a very reasonable assumption. It merely means that an individual is more likely to be classified as possessing defect type j if indeed it does possess this type of defect, than if it does not.

2. Moment Estimators

Our problem is to estimate the $D_{\underset{\sim}{g}}$'s from the observed values of the $Z_{\underset{\sim}{g}}$'s. The expected value of $D_{\underset{\sim}{g}}$ is

$$E[Z_{\underset{\sim}{g}}] = nN^{-1} \sum_{(\underset{\sim}{f})} D_{\underset{\sim}{f}} p_{\underset{\sim}{g}|\underset{\sim}{f}} \qquad (2)$$

and so the moment estimators, $\tilde{D}_{\underset{\sim}{f}}$, satisfy the 2^k equations

$$Z_{\underset{\sim}{g}} = nN^{-1} \sum_{(\underset{\sim}{f})} \tilde{D}_{\underset{\sim}{f}} p_{\underset{\sim}{g}|\underset{\sim}{f}} \qquad (3)$$

or, in matrix form

$$\underset{\sim}{Z} = nN^{-1} \underset{\sim}{P}\, \underset{\sim}{\tilde{D}} \qquad (3)'$$

where $\underset{\sim}{Z}$ and $\underset{\sim}{\tilde{D}}$ are 2^k-rowed vectors with elements $\{Z_{\underset{\sim}{g}}\}$ and $\{\tilde{D}_{\underset{\sim}{g}}\}$ respectively,

and $\underset{\sim}{P}$ is a $2^k \times 2^k$ matrix with $p_{\underset{\sim}{g}|\underset{\sim}{f}}$ as the element in the $\underset{\sim}{g}$-th row and $\underset{\sim}{f}$-th column. From (3)',

$$\tilde{\underset{\sim}{D}} = Nn^{-1}\underset{\sim}{P}^{-1}\underset{\sim}{Z} \tag{4}$$

where $\underset{\sim}{P}^{-1}$ is the left inverse of $\underset{\sim}{P}$. Since

$$E[\underset{\sim}{Z}] = nN^{-1}\underset{\sim}{P}\,\underset{\sim}{D}$$

it follows that $E[\tilde{\underset{\sim}{D}}] = \underset{\sim}{D}$, so $\tilde{\underset{\sim}{D}}$ is an unbiased estimator of $\underset{\sim}{D}$ ($E[\tilde{D}_{\underset{\sim}{g}}] = D_{\underset{\sim}{g}}$ for all $(\underset{\sim}{g})$.)

However it is possible for some of the $D_{\underset{\sim}{g}}$'s to be negative (though not all of them, because $\sum_{(\underset{\sim}{g})} \tilde{D}_{\underset{\sim}{g}} = N$). Similarly some (but not all) of the $D_{\underset{\sim}{g}}$'s might exceed N. This latter possibility occurs much less frequently.

For k=1 (a single type of defect) we have, of course

$$D_1 = N - D_0, \text{ and } E[Z_1] = N^{-1}n\{D_1p_1+(N-D)p_1'\}$$

$$= N^{-1}n\{Np_1' + D(p_1-p_1')\}$$

whence

$$\tilde{D}_1 = N(p_1-p'_1)^{-1}(n^{-1}Z_1-p_1') \tag{5}$$

Since $p_1 > p_1'$, $\quad \tilde{D}_1 < 0 \quad \text{if } Z_1 < np_1' \tag{6.1}$

$$\tilde{D}_1 > N \quad \text{if } Z_1 > np_1 \tag{6.2}$$

In practice, one would use a modified estimator such as

$$\tilde{D}_1^* = \begin{cases} 0 & \text{if } \tilde{D}_1 < 0 \\ \tilde{D}_1 & \text{if } 0 \le \tilde{D}_1 \le N \\ N & \text{if } N < \tilde{D}_1 \end{cases} \tag{7}$$

This estimator is not, in general, unbiased.

Table 1 shows values of

(i) $\Pr[\tilde{D}_1 < 0]$

(ii) $\Pr[\tilde{D}_1 > N]$

(iii) $\text{Bias } (\tilde{D}_1^*) = E[\tilde{D}_1^*] - \tilde{D}_1$

for a few sets of values of the parameters.

3. Distribution Theory

Conditional on the numbers $\underset{\sim}{Y} = \{Y_{\underset{\sim}{f}}\}$ of individuals with actual defect patterns $\{(\underset{\sim}{f})\}$ in the sample, the joint distribution of the 2^k numbers $\{W_{\underset{\sim}{g}|\underset{\sim}{f}}\}$ of the $Y_{\underset{\sim}{f}}$ individuals with pattern $(\underset{\sim}{f})$ which are assigned patterns $\{(\underset{\sim}{g})\}$ as a result of inspection is multinomial with parameters $(Y_{\underset{\sim}{f}};\{p_{\underset{\sim}{g}|\underset{\sim}{f}}\})$. For different $(\underset{\sim}{f})$ and $(\underset{\sim}{f}^*)$ and *any* $(\underset{\sim}{g})$, $(\underset{\sim}{g}^*)$ (whether or not $(\underset{\sim}{g}) \equiv (\underset{\sim}{g}^*)$) $W_{\underset{\sim}{g}|\underset{\sim}{f}}$ and $W_{\underset{\sim}{g}^*|\underset{\sim}{f}^*}$ are mutually independent binomial variables with parameters $(Y_{\underset{\sim}{f}}, p_{\underset{\sim}{g}|\underset{\sim}{f}})$, $(Y_{\underset{\sim}{f}^*}, p_{\underset{\sim}{g}^*|\underset{\sim}{f}^*})$ respectively. Since

$$Z_{\underset{\sim}{g}} = \sum_{(\underset{\sim}{f})} W_{\underset{\sim}{g}|\underset{\sim}{f}} \qquad (8)$$

the conditional joint distribution of $\underset{\sim}{Z}$, given $\underset{\sim}{Y}$, can be represented symbolically

$$\underset{\sim}{Z}|\underset{\sim}{Y} \frown \underset{(\underset{\sim}{f})}{*} \text{ Multinomial } (Y_{\underset{\sim}{f}};\{p_{\underset{\sim}{g}|\underset{\sim}{f}}\}) \qquad (9)$$

where * stands for "convolution" and "$\frown$" means "is distributed as".

To obtain the unconditional distribution of $\underset{\sim}{Z}$, this conditional distribution has to be compounded with respect to $\underset{\sim}{Y}$, which has a multivariate hypergeometric distribution with parameters n; $\underset{\sim}{D}$; N, so that

$$\Pr[\underset{\sim}{Y} = \underset{\sim}{y}] = \binom{N}{n}^{-1} \prod_{(\underset{\sim}{f})} \binom{D_{\underset{\sim}{f}}}{y_{\underset{\sim}{f}}} \quad (0 \le y_{\underset{\sim}{f}} \le D_{\underset{\sim}{f}}; \textstyle\sum_{(\underset{\sim}{f})} y_{\underset{\sim}{f}} = n) \qquad (10)$$

From (8) and (9), moments and product-moments of the $Z_{\underset{\sim}{g}}$'s can be evaluated (see [1], equations (17)).

4. Variances of the Estimators

From (4) the variance-covariance matrix of $\tilde{\underset{\sim}{D}}$ is

$$\underset{\sim}{\text{Var}}(\tilde{\underset{\sim}{D}}) = (Nn^{-1})^2 \underset{\sim}{P}^{-1} \underset{\sim}{\text{Var}}(\underset{\sim}{Z}) (\underset{\sim}{P}^{-1})^T \qquad (11)$$

(where the superfix (T) denotes 'transpose').

From [1] (equation (17))

$$\mathrm{var}(Z_{\underset{\sim}{g}}) = n\bar{p}_{\underset{\sim}{g}} - n\{(n-1)\overline{p^2_{\underset{\sim}{g}}} + (N-n)\bar{p}^2_{\underset{\sim}{g}}\}(N-1)^{-1} \tag{12.1}$$

$$\mathrm{cov}(Z_{\underset{\sim}{g}}, Z_{\underset{\sim}{g}^*}) = -n\{(n-1)\overline{p_{\underset{\sim}{g}} p_{\underset{\sim}{g}^*}} + (N-n)\bar{p}_{\underset{\sim}{g}}\bar{p}_{\underset{\sim}{g}^*}\}(N-1)^{-1} \tag{12.2}$$

where $p_{\underset{\sim}{g}} = N^{-1}\sum_{(\underset{\sim}{f})} D_{\underset{\sim}{f}} p_{\underset{\sim}{g}|\underset{\sim}{f}}$; $\overline{p^2_{\underset{\sim}{g}}} = N^{-1}\sum_{(\underset{\sim}{f})} D_{\underset{\sim}{f}} p^2_{\underset{\sim}{g}|\underset{\sim}{f}}$

and $\overline{p_{\underset{\sim}{g}} p_{\underset{\sim}{g}^*}} = N^{-1}\sum_{(\underset{\sim}{f})} D_{\underset{\sim}{f}} p_{\underset{\sim}{g}|\underset{\sim}{f}} p_{\underset{\sim}{g}^*|\underset{\sim}{f}}$.

5. Special Case: Uniform Accuracy of Inspection

If $p_j = p$ and $p_j = p'$ for all $j=1,\ldots,k$ - that is, the accuracy of inspection is the same for all types of defect - then

$$p_{\underset{\sim}{g}|\underset{\sim}{f}} = p^{s_{11}} p'^{s_{10}} (1-p)^{s_{01}} (1-p')^{s_{00}}$$

where s_{ab} = number of types (j) of defect for which $f_j = b, g_j = a$.

For k=2, we have

$$\underset{\sim}{P} = \begin{array}{c} (\underset{\sim}{g}) \\ 00 \\ 01 \\ 10 \\ 11 \end{array} \begin{array}{c} (\underset{\sim}{f}) = \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} (1-p')^2 & (1-p)(1-p') & (1-p)(1-p') & (1-p)^2 \\ p'(1-p') & p(1-p') & (1-p)p' & p(1-p) \\ p'(1-p') & (1-p)p' & p(1-p') & p(1-p) \\ p'^2 & pp' & pp' & p^2 \end{pmatrix} \end{array} \tag{13.1}$$

and

$$(p-p')^2 \underset{\sim}{P}^{-1} = \begin{pmatrix} p^2 & -p(1-p) & -p(1-p) & (1-p)^2 \\ -pp' & p(1-p') & (1-p)p' & -(1-p)(1-p') \\ -pp' & (1-p)p' & p(1-p') & -(1-p)(1-p') \\ p'^2 & -p'(1-p') & -p'(1-p') & (1-p')^2 \end{pmatrix} \tag{13.2}$$

Hence

$$\left.\begin{aligned}
\tilde{D}_{00} &= (Nn^{-1})(p-p')^{-2}\{p^2Z_{00}-p(1-p)(Z_{01}+Z_{10})+(1-p)^2Z_{11}\} \\
\tilde{D}_{01} &= (Nn^{-1})(p-p')^{-2}\{-pp'Z_{00}+p(1-p')Z_{01}+(1-p)p'Z_{10}-(1-p)(1-p')Z_{11}\} \\
\tilde{D}_{10} &= (Nn^{-1})(p-p')^{-2}\{-pp'Z_{00}+(1-p)p'Z_{01}+p(1-p')Z_{10}-(1-p)(1-p')Z_{11}\} \\
D_{11} &= (Nn^{-1})(p-p')^{-2}\{p^{12}Z_{00}-p'(1-p')(Z_{01}+Z_{10})+(1-p')^2Z_{11}\}
\end{aligned}\right\} \quad (14)$$

For example, if $p = 0.90$ and $p' = 0.95$ then

$$\tilde{D}_{00} = Nn^{-1}\{1.121Z_{00} - 0.125(Z_{01}+Z_{10}) + 0.014Z_{11})$$

$$\tilde{D}_{01} = Nn^{-1}(-0.062Z_{00}+ 1.183Z_{01} + 0.007Z_{10}- 0.131Z_{11})$$

$$\tilde{D}_{10} = Nn^{-1}(-0.062Z_{00}+ 0.007Z_{10} + 1.183Z_{10} -0.131Z_{11})$$

$$\tilde{D}_{11} = Nn^{-1}(0.003Z_{00} - 0.066(Z_{01}+Z_{10}) + 1.249Z_{11}\}$$

As is to be expected the major coefficient in the expression for D_{ab} is that of Z_{ab}, but the contributions of the other terms is not negligible.

For k =3 we have

$$
\begin{array}{c|c}
(\underset{\sim}{g}) \quad (\underset{\sim}{f}) = & \begin{array}{cccccccc} 000 & 001 & 010 & 100 & 011 & 101 & 110 & 111 \end{array} \\
\begin{array}{c} 000 \\ 001 \\ 010 \\ 100 \\ 011 \\ 101 \\ 110 \\ 111 \end{array} \quad \underset{\sim}{P} = & \begin{pmatrix}
(1-p')^3 & (1-p)(1-p')^2 & (1-p)(1-p')^2 & (1-p)(1-p')^2 & (1-p)^2(1-p') & (1-p)^2(1-p') & (1-p)^2(1-p') & (1-p)^3 \\
p'(1-p')^2 & p(1-p')^2 & (1-p)p'(1-p') & (1-p)p'(1-p') & p(1-p)(1-p') & p(1-p)(1-p') & (1-p)^2p' & p(1-p)^2 \\
p'(1-p')^2 & (1-p)p'(1-p') & p(1-p')^2 & (1-p)p'(1-p') & p(1-p)(1-p') & (1-p)^2p' & p(1-p)(1-p') & p(1-p)^2 \\
p'(1-p')^2 & (1-p)p'(1-p') & (1-p)p'(1-p') & p(1-p')^2 & (1-p)^2p' & p(1-p)(1-p') & p(1-p)(1-p') & p(1-p)^2 \\
p'^2(1-p') & pp'(1-p') & pp'(1-p') & (1-p)p'^2 & p^2(1-p') & p(1-p)p' & p(1-p)p' & p^2(1-p) \\
p'^2(1-p') & (1-p)p'^2 & (1-p)p'^2 & pp'(1-p') & p(1-p)p' & p^2(1-p') & p(1-p)p' & p^2(1-p) \\
p'^2(1-p') & (1-p)p'^2 & pp'(1-p') & pp'(1-p') & p(1-p)p' & p(1-p)p' & p^2(1-p') & p2(1-p) \\
p'^3 & pp'^2 & pp'^2 & pp'^2 & p^2p' & p^2p' & p^2p' & p^3
\end{pmatrix}
\end{array}
$$

$$
(p-p')^3\underset{\sim}{P}^{-1} = \begin{array}{c|c}
(\underset{\sim}{f}) = & \begin{array}{cccccccc} 000 & 001 & 010 & 100 & 011 & 101 & 110 & 111 \end{array} \\
& \begin{pmatrix}
p^3 & -p^2(1-p) & -p^2(1-p) & -p^2(1-p) & p(1-p)^2 & p(1-p)^2 & p(1-p)^2 & -(1-p)^3 \\
-p^2p' & p^2(1-p') & p(1-p)p' & p(1-p)p' & -p(1-p)(1-p') & -p(1-p)(1-p') & -(1-p)^2p' & (1-p)^2(1-p') \\
-p^2p' & p(1-p)p' & p^2(1-p') & p(1-p)p' & -p(1-p)(1-p') & -(1-p)^2p' & -p(1-p)(1-p') & (1-p)^2(1-p') \\
-p^2p' & p(1-p)p' & p(1-p)p' & p^2(1-p') & -(1-p)^2p' & -p(1-p)(1-p') & -p(1-p)(1-p') & (1-p)^2(1-p') \\
pp'^2 & -pp'(1-p') & -pp'(1-p') & -(1-p)p'^2 & p(1-p')^2 & (1-p)p'(1-p') & (1-p)p'(1-p') & -(1-p)(1-p')^2 \\
pp'^2 & -pp'(1-p') & -(1-p)p'^2 & -pp'(1-p') & (1-p)p'(1-p') & p(1-p')^2 & (1-p)p'(1-p') & -(1-p)(1-p')^2 \\
pp'^2 & -(1-p)p'^2 & -pp'(1-p') & -pp'(1-p') & (1-p)p'(1-p') & (1-p)p'(1-p') & p(1-p')^2 & -(1-p)(1-p')^2 \\
-p'^3 & p'^2(1-p') & p'^2(1-p') & p'^2(1-p') & -p'(1-p')^2 & -p'(1-p')^2 & -p'(1-p')^2 & (1-p')^3
\end{pmatrix}
\end{array}
$$

For example

$$\tilde{D}_{011} = (Nn^{-1})(p-p')^{-3}\{pp'^2Z_{000} - pp'(1-p')(Z_{001}+Z_{010}) - (1-p)p'^2Z_{100}$$

$$+p(1-p')^2Z_{011} + (1-p)p'(1-p')(Z_{101}+Z_{110}) - (1-p)(1-p')^2Z_{111}\} \qquad (16)$$

In these circumstances, if also $D_{\underset{\sim}{f}} = 2^{-k}N$ for all $(\underset{\sim}{f})$, then

$$\bar{p}_{\underset{\sim}{g}} = 2^{-k}(p+p')^m(2-p-p')^{k-m} \qquad (17.1)$$

where $m(\equiv m(\underset{\sim}{g}))$ is the number of 1's in $(\underset{\sim}{g})$;

$$\overline{p_{\underset{\sim}{g}}^2} = 2^{-k}(p^2+p'^2)^m\{(1-p)^2+(1-p')^2\}^{k-m} \qquad (17.2)$$

and $$\overline{p_{\underset{\sim}{g}}p_{\underset{\sim}{g}^*}} = 2^{-k}(p^2+p'^2)^{m_{11}}\{p(1-p')+(1-p)p'\}^{m_{01}+m_{10}}\{(1-p)^2+(1-p')^2\}^{m_{00}} \qquad (17.3)$$

where

$m_{ab}(\equiv m_{ab}(\underset{\sim}{g},\underset{\sim}{g}^*))$ is the number of types (j) of defect with $g_j = a$, $g_j^* = b$.

$(m_{11} + m_{10} + m_{01} + m_{00} = k)$.

The further specialization $p' = 1-p$, corresponding to a constant probability (1-p) of error, leads to further simplification. In particular,

$$\bar{p}_{\underset{\sim}{g}} = 2^{-k} \; ; \; \overline{p_{\underset{\sim}{g}}^2} = 2^{-k}\{p^2+(1-p)^2\}^k \text{ for all } (\underset{\sim}{g}) \qquad (18.1)$$

$$\overline{p_{\underset{\sim}{g}}p_{\underset{\sim}{g}^*}} = 2^{-k}\{p^2+(1-p)^2\}^{m_{00}+m_{11}}\{2p(1-p)\}^{m_{01}+m_{10}} \qquad (18.2)$$

Note that $(m_{00} + m_{11})$ equals the number of j's for which $g_j = g_j^*$; and $p^2 + (1-p)^2 = 1-2p(1-p)$. We will use the notation

$$m_{00} + m_{11} = \theta; \; p^2+(1-p)^2 = A; \; 2p(1-p) = 1-A = B \qquad (19)$$

From (12) and (18)

$$\text{var}(Z_{\underset{\sim}{g}}) = n\cdot 2^{-k}\left(1 - \frac{N-n}{N-1}\cdot 2^{-k} - \frac{n-1}{N-1}A^k\right) \qquad (20.1)$$

$$\text{cov}(Z_{\underset{\sim}{g}},Z_{\underset{\sim}{g}^*}) = -n\,2^{-k}\left(\frac{N-n}{N-1}\cdot 2^{-k} + \frac{n-1}{N-1}A^\theta B^{k-\theta}\right) \qquad (20.2)$$

As an example consider

$$\tilde{D}_{011} = (Nn^{-1})(2p-1)^{-6}\{p(1-p)^2(Z_{000}+Z_{101}+Z_{110})-p^2(1-p)(Z_{111}+Z_{001}+Z_{010}) + p^3Z_{011}-(1-p)^3Z_{100}\} \quad (21)$$

obtained from (16) by putting $p' = 1-p$.

We obtain from (20) and (21)

$$\text{var}(\tilde{D}_{011}) = \frac{1}{n}\cdot\frac{N^2}{8}\{\frac{A^3}{(2p-1)^6} - \frac{N-9n+8}{8(N-1)}\} \quad (22)$$

(Details are given in the Appendix. Note that

$$A^3(2p-1)^{-6} = \frac{1}{8}\{1+(1-2p)^{-2}\}^3.)$$

The proportion D_{011}/N ($=2^{-k}$ in this case) is estimated unbiasedly by $\tilde{D}_{011}/N$ and this estimator has variance

$$\frac{1}{64n}[\{1+(1-2p)^{-2}\}^3 - \frac{N-9n+8}{N-1}] \sim \frac{1}{64n}[\{1+(1-2p)^{-2}\}^3-1] \text{ if } n \ll N.$$

REFERENCES

[1] Kotz, S. and Johnson, N.L. (1983) Some distributions arising from faulty inspection with multitype defectives, and an application to grading, Commun. Statist. Theory Meth., 12 (To appear).

[2] Johnson, N.L. and Kotz, S. (1983). Multivariate distributions arising in faulty sampling inspection. Proc. 6th Internat. Symp. Multiv. Analysis, Pittsburgh (Ed. P.R. Krishnaiah) Amsterdam, North Holland (To appear).

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APPENDIX: Derivation of the Variance of D_{011}

From (20) and (21)

$$(Nn^{-1})^{-2}(2p-1)^6 \mathrm{var}(\tilde{D}_{011}) = \frac{1}{8}nS_2 - \frac{n(N-n)}{64(N-1)}S_1^2 - \frac{n(n-1)}{8(N-1)}[A^3S_2 +$$
$$2\{p^2(1-p)^4(3AB^2)+p^4(1-p)^2(3AB^2)$$
$$-p^3(1-p)^3(B^2+2AB^2+4AB^2+2B^3)-p^3(1-p)^3B^3$$
$$+p^4(1-p)^2(3AB^2)+p^2(1-p)^4(3AB^2)$$
$$-p(1-p)^5(3A^2B)-p^5(1-p)(3A^2B)\}]$$
$$= \frac{1}{8}nS_2 - \frac{n(N-n)}{64(N-1)}S_1^2 - \frac{n(n-1)}{8(N-1)}[A^3S_2$$
$$- 6\{p(1-p)^5+2p^3(1-p)^3+p^5(1-p)\}A^2B$$
$$+12\{p^2(1-p)^4+p^4(1-p)^2\}AB^2-8p^3(1-p)^3B^3]$$
$$= \frac{1}{8}nS_2 - \frac{n(N-n)}{64(N-1)}S_1^2 - \frac{n(n-1)}{8(N-1)}(A^3S_2 - 3A^4B^2+3A^2B^4-B^6)$$

where S_2 = sum of squares of coefficients of Z's in $(Nn^{-1})^{-1}(2p-1)^3\tilde{D}_{011}$

$= 3p^2(1-p)^4+3p^4(1-p)^2 + p^6+(1-p)^6 = A^3$

and S_1 = sum of coefficients of Z's in $(Nn^{-1})^{-1}(2p-1)^3\,\tilde{D}_{011}$

$= 3p(1-p)^2-3p^2(1-p)+p^3-(1-p)^3 = (2p-1)^3$

Noting that since A+B = 1,

$$A^3S_2 - 3A^4B^2 + 3A^2B^4-B^6 = (A^2-B^2)^3 = (A-B)^3 = (2p-1)^6$$

we obtain

$$\mathrm{var}(\tilde{D}_{011}) = \frac{1}{n}\cdot\frac{N^2}{8}\left\{\frac{A^3}{(2p-1)^6} - \frac{N-9n+8}{8(n-1)}\right\} = \frac{1}{n}\cdot\frac{N^2}{64}\left[\{1+(2p-1)^{-2}\}^3-1+\frac{8(n-1)}{N-1}\right] \quad (22)$$

The *proportion* D_{011}/N is estiamted unbiasedly by $\tilde{D}_{011}/N$; this estimator has variance

$$\frac{1}{64n}\left[\{1+(2p-1)^{-2}\}^3 - 1+\frac{8(n-1)}{N-1}\right]. \quad (22)'$$

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Unbiased estimators of the number of individuals in a lot possessing various patterns of types of defects are constructed. Explicit formulas are given for cases of two and three types of defect. Application of the formulas requires knowledge of the probabilities of various kinds of errors in the inspection process.

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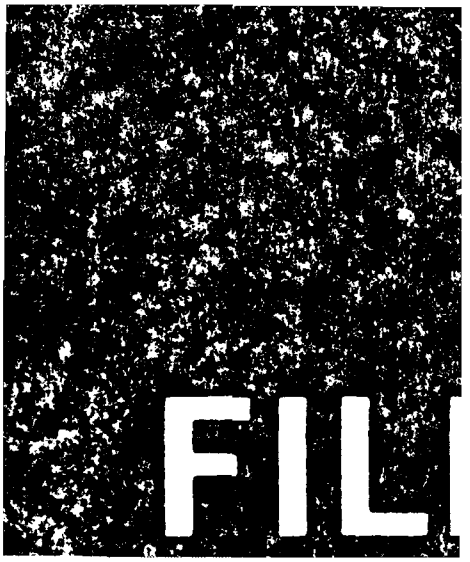
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